the addition of an element in combination, as the velocity of oxygen is increased, by combining it with carbon without change of volume, in carbonic acid gas.

It did not enter into the plan of the author to investigate the passage of gases through tubes of great diameter, and to solve pneumatic problems of actual occurrence, such as those offered in the distribution of coal-gas by pipes. But he states that the results must be similar, with truly elastic gases such as air and carburetted hydrogen, whether the tubes be capillary or several inches in diameter, provided the length of the tube be not less than 4000 times its diameter, as in the long glass capillaries of his experiments. The small propulsive pressure applied to coal-gas is also favourable to transpiration, as well as the great length of the mains; and he therefore would expect the distribution of coal-gas in cities to exemplify approximately the laws of gaseous transpiration. The velocity of coal-gas should be 1·575, that of air being 1, under the same pressure. And with a constant propulsive pressure in the gasometer, the flow of gas should increase in volume with a rise of the barometer or with a fall in temperature, directly in proportion to the increase of its density from either of these causes.

These laws, it will be observed, are entirely different from those which direct the passage of gases through an aperture in a thin plate, or their flow into a vacuum as it is usually said, and could not be deduced, like the latter, from our speculative ideas respecting the elastic fluids.

11. "On the Automatic Registration of Magnetometers and Meteorological Instruments by Photography."—No. III. By Charles Brooke, M.B., F.R.S.

The author describes the construction of an apparatus for registering the variation of the thermometer and psychrometer on one sheet of paper. As in the apparatus for registering the vertical force magnetometer, described in a former paper, the photographic paper is placed between two concentric cylinders, placed with the axis vertical, and carried round on a revolving plate or turn-table by the hour-hand of a time-piece, which makes half a revolution in twenty-four hours; thus each half of the paper presents a record of the variation of one instrument during twenty-four hours. The scales of the instruments are continuously impressed on the paper by placing fine wires opposite each degree across the aperture through which the light falls on the stem; the light transmitted by the empty bore is intercepted by these wires, and the darkened portion of the paper is marked by a series of parallel pale lines corresponding to each degree: thus the distortion of the scale arising from the varying direction of the pencils of light is corrected. Every tenth degree is marked by a coarser wire, and therefore a broader line, as also the points 32°, 54°, 76°, 98°; one at least of these points will occur on each register, and the position of the extra broad line serves to identify the part of the scale to which the register relates.

An alteration in the mode of adjusting the wick of the camphine lamps described in a former paper is mentioned, by which the chance

of smoking is considerably diminished; likewise the successful application of naphthalized gas, and of an oil-lamp, to photographic registration.

The paper concludes with the description of a new method of determining the scale and temperature coefficients of the force magnetometers, by which a greater degree of accuracy is presumed to be attained than by the methods ordinarily employed. Two magnets, designed for self-registering instruments for the observatories at Cambridge and Toronto, having been submitted to this method, gave consistent results which indicate the law of the temperature coefficient to be sensibly different from that which has hitherto been assumed.

12. "On certain Properties of the Arithmetical Series whose ultimate differences are constant." By Sir Frederick Pollock, Lord Chief-Baron of the Exchequer, F.R.S. &c.

This paper professes to investigate certain properties of the series of whole numbers whose ultimate differences are constant, and incidentally to treat of Fermat's theorem of the polygonal numbers, and some other properties of numbers.

Its object is to show that the same (or an analogous) property which Fermat discovered in the polygonal numbers belongs to other series of the same order, also to all series of the first order, and probably to all series of all orders. It also proposes to prove the first case of Fermat's theorem (that is of the triangular numbers) from the second case of the squares (which had not before been done), and to dispense with the elaborate proof of Legendre (Théorie des Nombres), finally, to prove all the cases by a method different from that either of Lagrange, Euler, or Legendre.

It is first shown that an analogous property belongs to all series of the first order (viz. common arithmetical series). The following propositions are then proved as the basis of future reasoning:—

1. Every triangular number greater than 6 is composed of 3 triangular numbers.

2. Every triangular number greater than 3 is composed of 4 triangular numbers.

3. Any triangular number may be expressed by the form (a^2+a+b^2).

4. The sum of any two triangular numbers may be expressed by the same form.

5. Every number above 7 is the sum of *four* triangular numbers *exactly*.

6. Every number above 29 is the sum of *three* triangular numbers *exactly*.

7. Every multiple of 8 is composed of eight odd squares, and the sum of any 8 odd squares is a multiple of 8.

8. The following general theorem is then proved:—

If p be any odd square, then

$$Ap^x + Bp^y + Cp^z + Dp^w + \text{ &c.}$$

will equal 8 odd squares, if

$$A + B + C + D + \text{, &c.}$$